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Relativistic properties of spin and pion electromagnetic structure

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Abstract: The role of the Wigner spin rotation effects in the electromagnetic structure of pion is investigated. Pion is considered as quark-antiquark system in the framework of the instant form relativistic hamiltonian dynamics. It is shown that the spin rotation effects give the important contribution to the pion mean square radius, to the pion form factor at the intermediate momentum transfers and to the form-factor asymptotic behaviour. The calculations are performed with the different model wave functions. It is shown that the contribution of the spin rotation depends weakly on the model for the interaction of quarks in pion.

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The increasing interest for spin physics problems is caused by the spin crisis problem, by a number of precise polarization experiments performed, and by the experiments presently on line in JLab [1].

In this connection it is important to develop the theoretical analysis of the spin properties of hadrons and hadron systems, and particularly, of the spin relativistic properties.

The subject of the present work is the investigation of the relativistic Wigner spin rotation (SR) as related to the electromagnetic form factor of $\pi-$meson which is considered as quark-antiquark composite system.

There are two main different approaches to the relativistic theory of composite systems. The first one is that of field theory and is based on the summation of a class of perturbation theory diagrams, on the equation of Bethe-Salpeter type, and on the different quasipotential reduction of this equation (see e.g. [2]–[6]). Different variants of dispersion approaches [7]–[9] are naturally connected with this method. The second approach is based on the direct realization of the Poincaré group algebra, in other words, of the relativistic invariance condition in the few body Hilbert space.

This second approach is usually called relativistic hamiltonian dynamics (RHD) and it follows the classic Dirac paper [10]. There are three types of dynamics connected with different ways of interaction inclusion (or with different ways of dividing the generators into two groups: that of kinematical generators and that of hamiltonians [11]–[13]): the light front dynamics, the instant form dynamics and the point form dynamics. The most popular is the first one — the light front dynamics (see e.g. [14]–[23]), which has many advantages, one being the minimal number of hamiltonians. The detailed description of the characteristics and the possibilities of the approach can be found in the references above. The approaches using the diagram technique on the light front should be mentioned [24, 25]. The connection between different dynamics forms is discussed in [13, 26, 27]. The light front dynamics is applied to the calculation of pion electromagnetic structure in ref. [17]–[23].

The RHD development left other forms of dynamics, and among them the formerly familiar instant form dynamics, in the shadow. However some works were published recently where the instant form dynamics [28]–[31] and the point form dynamics [32, 33] were used.

We believe that it is the instant form that is suitable for the investigation of SR because this effect appears in the instant form dynamics quite naturally. We shall use the variant of the instant form RHD developed by the authors [28]–[31].

The first step in the description of the electromagnetic structure of pion is the choice of the basis in the Hilbert space of states adequate to the instant form dynamics. Let us consider a system of two free spin 1/2 particles. The states of the system are described by the vectors which will be used as a basis:

$$|\,\vec{P},\sqrt{s},J,l,S,m_J\rangle\,. \tag{1}$$

Here $P=(p_1+p_2)$ is the total momentum, $P^2=s$, $\sqrt{s}$ is the invariant system mass, J is the total angular momentum, l is the orbital moment, S is the total spin, m_J is the projection of the total angular momentum.

The inclusion of the interaction into the Poincaré algebra of observables is produced in the line with general prescription of RHD [11]–[13].

Now we shall follow the general prescriptions of RHD for the case of instant form (see e.g. [11]). We include the interaction in the free system mass square operator: $P^2=s\to\hat{M}_I^2=s+4M\hat{V}$. The complete set of commuting operators in the instant form dynamics is: the total momentum operator $\hat{\vec{P}}$, the total angular momentum square operator $\hat{J}^2$, the total angular momentum projection operator $\hat{J}_3$ and operator $\hat{M}_I^2$. In the instant form dynamics the operators $\hat{\vec{P}}$, $\hat{J}^2$, $\hat{J}_3$ coincide with those of noninteracting system and are diagonal in the basis (1). The wave function of the system of interacting particles is the eigenfunction for the mentioned set of operators. In the basis (1) it has the form:

$$\langle\vec{P}\,',\sqrt{s'},\,J',\,l',\,S',\,m'_J|\,p_c\rangle=N_c\delta(\vec{P}\,'-\vec{p}_c)\delta_{JJ'}\delta_{m_Jm'_J}\delta_{ll'}\delta_{SS'}\,\varphi^J_{lS}(k)\,. \tag{2}$$

Here $s=4M^2{+}2ME$, $k^2=EM/2$, E is the energy of particles motion, $\varphi^J_{lS}(k)=\varphi(k)$ is the wave function of the internal motion of quarks in pion ($J=l=S=0$), N_c is a factor due to normalization. Particular form of N_c may not be specified. As is known in the case of RHD [11], the eigenvalue problem for the operator $\hat{M}_I^2$ gives an equation of the form of nonrelativistic Schrödinger equation, so that one can use for $\varphi(k)$ one of the phenomenological wave function. $\varphi(k)$ is normalized with the account of relativistic density of states [29]:

$$\int\frac{dk}{2\sqrt{k^2+M^2}}\,\varphi^2(k)=1\,, \tag{3}$$

with

$$\varphi(k)=\sqrt[4]{4(k^2+M^2)}\,k\,u(k)\,,\qquad\int dk\,k^2\,u^2(k)=1\,. \tag{4}$$

Here $u(k)$ is a nonrelativistic wave function. The eq. (3) gives the normalization of the pion charge form factor $F_\pi(0)=1$.

To calculate pion form factor, we use the following wave functions for the ground state of quark-antiquark system:

1. Harmonic oscillator (HO) wave function (see e.g. [17]):

$$u(k)=N_{HO}\exp\left(\frac{-k^2}{2\,b^2}\right)\,. \tag{5}$$

2. Power-law (PL) wave function (see e.g. [20]):

$$u(k) = N_{PL} \left(\frac{k^2}{b^2} + 1 \right)^{-n}, \qquad n = 2, 3\,. \tag{6}$$

3. The wave function with linear confinement and Coulomb-like behavior at small distances [34]:

$$u(r) = N_T \exp(-\alpha r^{3/2} - \beta r)\,, \tag{7}$$

$$\alpha = \frac{2}{3}\sqrt{M\,a}\,, \qquad \beta = \frac{M\,b}{2}\,.$$

In eq. (7), a and b are the parameters of linear and Coulomb parts of the potential, respectively.

Let us consider the pion current matrix element:

$$\langle P_\pi \mid j_\mu \mid P'_\pi \rangle = (P_\pi + P'_\pi)_\mu \, F_\pi(Q^2)\,.$$

Here $F_\pi(Q^2)$ is electromagnetic pion form factor.

In our approach the following expression for the $F_\pi(Q^2)$ can be obtained in the relativistic impulse approximation:

$$F_\pi(Q^2) = \int d\sqrt{s}\, d\sqrt{s'}\, \varphi(k)\, g_0(s, Q^2, s')\, \varphi(k')\,. \tag{8}$$

The details of the derivation of equation (8) can be found in ref. [30]. The function $g_0(s, Q^2, s')$ is an invariant function to be called two-particle free form factor, which determines the matrix element of electromagnetic current of two free quarks.

For the calculation of $g_0(s, Q^2, s')$ we need the electromagnetic current matrix element of two free quarks in the basis (1) in the channel with pion quantum numbers $J = l = S = 0$. Following general principles of relativistic invariance and using the current conservation law we can write the matrix element in the form:

$$\left\langle \vec{P}, \sqrt{s}, J, l, S, m_J \right| j^0_\mu \left| \vec{P}', \sqrt{s'}, J', l', S', m_{J'} \right\rangle = A_\mu(s, Q^2, s') g_0(s, Q^2, s') \tag{9}$$

with

$$A_\mu = \left(\frac{1}{Q^2} \right) \left[(s - s' + Q^2) P_\mu + (s' - s + Q^2) P'_\mu \right], \qquad Q^2 = -(P - P')^2$$

The basis (1) is adequate to an irreducible Poincaré group representation. One can decompose it in the basis realizing the direct production of two irreducible representations: $|\vec{p}_1\,, m_1; \vec{p}_2, m_2\rangle = |\vec{p}_1, m_1\rangle \otimes |\vec{p}_2, m_2\rangle$. Here p_1, p_2 are particle momenta, m_1, m_2 are spin projections, $|\vec{p}, m\rangle$ is one particle state vector. This decomposition is just Clebsch-Gordan decomposition of Poincaré group [35, 36]. Let us note, that Wigner spin rotation parameters enter Clebsch-Gordan coefficients. Performing

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Clebsch-Gordan decomposition in (9) one can obtain $g_0(s,Q^2,s')$ in terms of usual one-particle Sachs form factors $G_E(Q^2)$, $G_M(Q^2)$. Free two-particle form factor has the form:

$$g_0(s,Q^2,s') = \frac{(s+s'+Q^2)Q^2}{2\sqrt{(s-4M^2)(s'-4M^2)}}\frac{\theta(s,Q^2,s')}{[\lambda(s,-Q^2,s')]^{3/2}}\times \tag{10}$$
$$\times\frac{1}{\sqrt{1+\frac{Q^2}{4M^2}}}\Big\{(s+s'+Q^2)\left(G_E^q(Q^2)+G_E^{\bar q}(Q^2)\right)\cos\left(\omega_1+\omega_2\right)+$$
$$+\frac{1}{M}\xi(s,Q^2,s')\left(G_M^q(Q^2)+G_M^{\bar q}(Q^2)\right)\sin\left(\omega_1+\omega_2\right)\Big\}.$$

Here $\lambda(a,b,c)=a^2+b^2+c^2-2(ab+ac+bc)$, $\xi=\sqrt{ss'Q^2-M^2\lambda(s,-Q^2,s')}$, ω_1 and ω_2 are the Wigner rotation parameters:

$$\omega_1 = \arctan\frac{\xi(s,Q^2,s')}{M[(\sqrt{s}+\sqrt{s'})^2+Q^2]+\sqrt{ss'}(\sqrt{s}+\sqrt{s'})}$$
$$\omega_2 = \arctan\frac{(2M+\sqrt{s}+\sqrt{s'})\xi(s,Q^2,s')}{M(s+s'+Q^2)(2M+\sqrt{s}+\sqrt{s'})+\sqrt{ss'}(4M^2+Q^2)}\,, \tag{11}$$

$\theta(s,Q^2,s')=\vartheta(s'-s_1)-\vartheta(s'-s_2)$, ϑ is step function and

$$s_{1,2}=2M^2+\frac{1}{2M^2}(2M^2+Q^2)(s-2M^2)\mp\frac{1}{2M^2}\sqrt{Q^2(Q^2+4M^2)s(s-4M^2)}\,.$$

Note that the magnetic form factor contribution to the eq. (10) is due to the spin rotation effect only. In the case of point particles $G_E^q+G_E^{\bar q}=G_M^q+G_M^{\bar q}=1$.

The quark structure is described by the following expressions:

$$G_E^q(Q^2)=e_q\,f(Q^2)\,,\quad G_M^q(Q^2)=(e_q+\kappa_q)\,f(Q^2)\,. \tag{12}$$

Here e_q is the charge of quark, κ_q is the quark anomalous magnetic moment in natural units, $f(Q^2)$ is the quark form factor.

One can see from eqs. (8), (10) and (12) that the SR is responsible for the contribution of the quark magnetic moment to the charge pion form factor. The neglect of the SR effect ($\omega_1=\omega_2=$0 in eq. (10)) leads to the elimination of this contribution. Let us note that the pion charge form factor (8) contains the contribution of the quark magnetic moment contrary to the case of light front dynamics [17].

For the calculation of the pion form factor from eqs. (8) and (10) it is necessary to specify the quark form factors. In the framework of the widely used point-quark approximation one has:

$$G_E^q+G_E^{\bar q}=G_M^q+G_M^{\bar q}=1\,. \tag{13}$$

However, there are strong arguments against this approximation. The model independent Gerasimov sum rules [37] indicate the existence of anomalous magnetic moments of constituent quarks. Explicit form of the quark form factor $f_q(Q^2)$ remains unknown. But it is possible to obtain some constraints on the quark form factor in our approach. To do this let us consider the asymptotic behaviour of charge pion form factor. In fact, the form factor $g_0(s,Q^2,s')$ in (9) is a regular distribution (generalized function) and is a linear continuous functional on a test functions space [38]. So pion form factor (8) is the functional given by $g_0(s,Q^2,s')$ on the space of functions $\psi(s,s') = \varphi(k(s))\,\varphi(k(s'))$. Thus, to obtain the asymptotic behavior of the pion form factor, one has to evaluate the asymptotic behavior at $Q^2 \to \infty$, $M \to 0$ of the generalized function $g_0(s,Q^2,s')$.

The constituent-quark mass M and the momentum-transfer square Q^2 are the parameters of the generalized function $g_0(s,Q^2,s')$. So the limits $Q^2 \to \infty$ and $M \to 0$ are to be considered in the generalized sense (in the sense of distributions), as the limits of the functional.

To obtain the asymptotic expansion of the generalized function we shall make the following assumptions.

1. Let the generalized function be given on the space R^2.

2. For the test function space let us take as usually the space $\mathcal{S}(R^2)$ of infinitely differentiable functions, decreasing at $|\vec{s}| \to \infty$ ($\vec{s} = (s,s')$) with all its derivatives faster than any degree of $1/|\vec{s}|$ [38].

3. Let us define the functional giving the regular generalized function by

$$\langle \tilde{g}_0(s,Q^2,s')\,,\phi(s,s')\rangle = \int d\mu(s,s')\,\tilde{g}_0(s,Q^2,s')\,\phi(s,s')\,. \tag{14}$$

   Here

$$\tilde{g}_0(s,Q^2,s') = 16\,\vartheta(s-4M^2)\,\vartheta(s'-4M^2)\,g_0(s,Q^2,s') \tag{15}$$

   where

$$d\mu(s,s') = \sqrt[4]{ss'}\,d\mu(s)\,d\mu(s')\,, \qquad d\mu(s) = \frac{1}{4}k\,d\sqrt{s}\,,$$

   and $\vartheta(x)$ is the standard step function. If $\phi(s,s') = u(k(s))\,u(k'(s'))$ then the functional (14) has the form (8).

To estimate the asymptotics of the integral in (14) let us realize the following steps [39]:

1. Let us transform the integral in s' to the integral on the segment [0,1].

2. Let us introduce the function $\tilde{\phi}(s,s')$ which is monotone decreasing function of the variable s when s' is fixed and of s' when s is fixed ($s,s' > 0$) and such that

$$|\phi(s,s') - \tilde{\phi}(s,s')| \to 0\,, \qquad |\vec{s}| \to \infty \tag{16}$$

3. Let us introduce two dimensionless parameters and let us realize the limiting procedure keeping $\eta = Q^2/b^2 \to \infty$, $\xi = b/2M \to \infty$.

4. In the integral in s let us introduce the new variable $y = E/2M$ (see (2)) and let us separate explicitly the dependence on the parameter η introduced above.

5. Let us restrict ourselves by the leading term of the integrand in the functional (14) when $\eta \to \infty$. This can be done because the integrals do converge uniformly.

As the result we obtain the following estimate for the functional (14) when $\eta \to \infty$:

$$\begin{aligned}\langle \tilde{g}_0(s, Q^2, s'), \phi(s, s')\rangle \sim \frac{1}{16}\sqrt[4]{\eta}\left(\frac{2M}{b}\right)^{5/2} b^3 \int\limits_0^\infty dy \frac{\sqrt{y}}{(1+y)^{1/4}} \times \\ \times \int\limits_0^1 dt \frac{\tilde{\phi}(s, \tilde{s})}{a(t)^{1/4}} \frac{1}{\left[\sqrt{y}(2t-1) + \sqrt{1+y}\right]^2} \times \\ \times \Big\{ \left[\sqrt{y}(2t-1) + \sqrt{1+y}\right] \cos(\tilde{\omega}_1 + \tilde{\omega}_2) + \\ + 2\sqrt{y}\sqrt{t(1-t)} \sin(\tilde{\omega}_1 + \tilde{\omega}_2)\Big\}. \qquad (17)\end{aligned}$$

In (17), $\tilde{\omega}_{1,2}$ are the leading terms in the expansion of the spin rotation parameters which enter $g_0(s, Q^2, s')$,

$$\tilde{s} = \eta\, b^2\, a(t) = \eta\, b^2 \left(1 + \frac{s - 4M^2}{2M^2} + \frac{1}{2M^2}(2t-1)\sqrt{s(s-4M^2)}\right).$$

To estimate the inner integral in (17) let us separate explicitly the parameter ξ, introduced above and let us obtain the asymptotics of the integral when $\xi \to \infty$. The leading term has the following form:

$$J(\eta, \xi, y) \sim \sqrt{\frac{b}{2\xi}} \frac{1}{\sqrt[4]{s - 4M^2}} \int\limits_0^1 dt\, \tilde{\phi}(s, \tilde{s}_0(t))\, f(t, y), \qquad (18)$$

$$f(t, y) = \frac{t^{1/4}}{\left[\sqrt{y}(2t-1) + \sqrt{1+y}\right]^2} \left\{\sqrt{y}(2t-1) + \sqrt{1+y} + 2\sqrt{y}(1-t)\right\}. \qquad (19)$$

Here

$$\tilde{s}_0(t) = 4\,\eta\,\xi^2\,(s - 4M^2)\left(1 + \frac{b^2}{2(s-4M^2)\,\xi^2}\right) t. \qquad (20)$$

It is possible to show that it is a small ε-neighborhood of the point $t = 0$ that gives the main contribution to the asymptotic expansion of the integral (19) (see [39]).

After expansion of $\tilde{\phi}(s, \tilde{s}_0)$, $\ln \tilde{\phi}(s, \tilde{s}_0)$ in the ε-vicinity of the point $t = 0$ and simple transformations we reduce the integral (18) to the standard form of [40, theorem 7.1]. The application of this theorem gives the following (up to preasymptotic factor) asymptotic of functional (14):

$$\langle \tilde{g}_0(s, Q^2, s')\,, \phi(s, s')\rangle \sim \left\langle \frac{1}{Q^2}\, g(s, s')\,, \phi(s, s') \right\rangle . \tag{21}$$

Finally, we proved that in the sense of distributions the following asymptotic estimate is valid:

$$g_0(s, Q^2, s') \sim \frac{1}{Q^2}\, g(s, s')\,, \qquad Q^2 \to \infty\,, \qquad M \to 0\,. \tag{22}$$

So, the comparison of eqs. (8) and (22) gives the asymptotic behaviour of pion form factor independently on the model of interaction of the constituent quarks in pion:

$$F_\pi(Q^2) \sim \frac{1}{Q^2}\,, \qquad Q^2 \to \infty\,, \qquad M \to 0\,. \tag{23}$$

In our approach this result coinciding with the prediction of QCD follows directly from relativistic kinematics which is valid for all energies. Let us emphasize that if we neglect, for example, the relativistic spin rotation effect (if we let the rotation parameters $\omega_{1,2}$ in $g_0(s, Q^2, s')$ be zero) then the asymptotic behavior of the generalized function in (22) is immediately changed.

We use the model (5) to demonstrate the role of relativism and, in particular, the role of the relativistic spin rotation effect. Explicit form of wave function (5) gives the possibility to use the [40, theorem 7.1] for the asymptotic expansion of (8) at $Q^2 \to \infty$ directly:

$$F_\pi(Q^2) \sim 4\sqrt{2}\frac{M}{Q}\,\exp\left(-\frac{Q\,M}{4b^2}\right)\left(1 + 8\,\frac{b^2}{Q\,M}\right) . \tag{24}$$

First and second addends in (24) are leading terms of asymptotics expansion at $Q^2 \to \infty$ of the parts of the integral (8), (10) which containe $\cos(\omega_1 + \omega_2)$ and $\sin(\omega_1 + \omega_2)$ respectively. The ultrarelativistic limit $M \to 0$ gives

$$F_\pi(Q^2) \sim 32\sqrt{2}\frac{b^2}{Q^2}\,, \tag{25}$$

while without relativistic spin rotation ($\omega_1 = \omega_2 = 0$ in eq. (10)) one has:

$$F_\pi(Q^2) \sim 4\sqrt{2}\frac{M}{Q}\,. \tag{26}$$

The contribution of the relativistic spin rotation is essential. If we neglect this contribution, we do not obtain correct (i.e. coinciding with QCD) asymptotic behavior.

It should be remarked that the asymptotic behaviour $\sim Q^{-2}$ as $Q^2 \to \infty$, $M \to 0$ in harmonic oscillator model (5) was obtained for light-front dynamics [41] and point dynamics [33], too.

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To clear up the role of relativism in our calculation, let us write the nonrelativistic asymptotic behavior obtained from the asymptotic estimate of nonrelativistic limit of eq. (8) in the case of the model (5):

$$F_\pi(Q^2) \sim \exp\left(-\frac{Q^2}{16\,b^2}\right). \tag{27}$$

The nonrelativistic asymptotic estimate gives gaussian decrease of the form factor. The comparison of (25) and (26) with (27) emphasizes the role of relativism. Let us note, that in (25)–(27) the explicit form of the normalization constant in (5) is taken into account: $N_{HO} = (4\,/\sqrt{\pi}\,b^3)^{1/2}$. Hence, relativistic effect of spin rotation provides the correct (e.g., coinciding with the QCD prediction) asymptotic of pion form factor.

In the case of non-point quarks the asymptotic estimate has the following form:

$$F_\pi(Q^2) \sim Q^{-2}\,\tilde{f}(Q^2)\,. \tag{28}$$

Here $\tilde{f}(Q^2)$ is the asymptotic behavior of quark form factor (12). To obtain the coinciding with QCD asymptotic behavior from (28), it is necessary to put in (28):

$$\tilde{f}(Q^2) \sim \frac{1}{\ln Q^2}\,. \tag{29}$$

So, the condition for eq. (28) to coincide with the prediction of QCD imposes the constrains on the explicit form of quark form factor. The simplest form, which satisfies the eq. (29), is the following:

$$f_q(Q^2) = \frac{1}{1 + \ln(1 + \langle r_q^2\rangle Q^2/6)}\,. \tag{30}$$

Here $\langle r_q^2\rangle$ is the mean square radius (MSR) of quark.

Let us discuss now the role of the SR in the calculation of pion form factor performed in the region of finite Q^2. For the calculation we use the eqs. (8), (10), (12) and (30). We have calculated the pion elastic form factor and MSR of pion in two cases:

(a) Relativistic calculation without spin rotation ($\omega_1 = \omega_2 = 0$ in eq. (10)) $F_R(Q^2)$, $\langle r^2\rangle_R^{1/2}$;

(b) Relativistic calculation with SR $F_{R+SR}(Q^2)$, $\langle r^2\rangle_{R+SR}^{1/2}$.

In our calculations we have fixed the masses of u- and d-quarks to be equal to 0.25 GeV. These values are usually used in relativistic calculations. For the parameters $\langle r_q^2\rangle$ from (30) we shall use the relation $\langle r_q^2\rangle \simeq 0.3/M^2$ between the MSR and the mass of the constituent quark [22, 23, 42]. Parameters b in the models (5) and (6)

Models	Parameters of wave functions $b\,,a$	$\langle r^2\rangle^{1/2}_R$	$\langle r^2\rangle^{1/2}_{R+SR}$
(5)	0.325	0.795	0.662
(6) n=2	0.396	0.786	0.669
(6) n=3	0.571	0.788	0.669
(7)	0.083	0.790	0.669
Experiment	0.657∓0.012 [43]		

Table 1: The results of calculation of MSR of pion without spin rotation ($\langle r^2\rangle^{1/2}_R$) and with spin rotation ($\langle r^2\rangle^{1/2}_{R+SR}$) for the models (5)–(7). The parameters in the models specified by equations (5) and (6) (b) are in GeV, while the parameter in the model specified by (7) (a) is in GeV^2. $M = 0.25\,\mathrm{GeV}$, $\kappa_u + \kappa_{\bar{d}} = -0.09$, $\langle r^2_q\rangle \simeq 0.3/M^2$. Parameter $b = 0.7867$ is used for the model (7).

or parameter a in the model (7) are fixed by the fit to the experimental MSR of pion [43]. We have used $b = 0.7867$ for the model (7) which means that the strong coupling constant is equal to 0.59 on the light meson mass scale.

Let us remark that these values of parameters give the lepton decay constant close to its experimental value [31].

The sum of anomalous magnetic moments of $u-$ and $\bar{\mathrm{d}}$-quarks $\kappa_u + \kappa_{\bar{d}}$ is a free parameter in the region $[-0.1; 0.1]$.

Let us consider the region of small Q^2, i.e. the region of MSR of pion. The results of the calculation for $M = 0.25\,\mathrm{GeV}$, $\kappa_u + \kappa_{\bar{d}} = -0.09$ are represented in table 1. From this table one can see that the experimental value of the pion MSR [43] $\langle r^2\rangle^{1/2}_{exp} = 0.657 \mp 0.012 fm$ can be obtained in the case (b). The SR contribution to MSR of pion is about 30%, which is an order of magnitude greater than the experimental data error. The value of this contribution is weakly dependent on the model.

Let us discuss the role of SR in the region of intermediate Q^2. For the description of the contribution of SR we introduce a function $W(Q^2)$:

$$W(Q^2) = \frac{F_{R+SR}(Q^2) - F_R(Q^2)}{F_{R+SR}(Q^2)}\,. \tag{31}$$

The function $W(Q^2)$ was calculated with the different model wave functions (5)–(7). The parameters of these wave functions are presented in table 1. Let us remark that $W(Q^2)$ is independent on the explicit form of the quark form factor $f_q(Q^2)$ (30). The results of the calculation are shown on the figure 1. The weak model dependence of these results is noteworthy. One can see that SR contribution varies from $\sim 10\%$ at $Q^2 \simeq 0.3\,(\mathrm{GeV/c})^2$ to $\sim 25\%$ at $Q^2 = 5\,(\mathrm{GeV/c})^2$.

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Let us remark that our calculations for all models (5)–(7) with parameters from table 1 are in good agreement with the existing experimental data in the region $Q^2 \leq 6\,(\mathrm{GeV/c})^2$.

To conclude, our main results are:

1. The contribution of SR to MSR of pion is bout 30% for any model of quark interaction.

2. The relativistic SR effect gives positive contribution (of about 10-25%) to the pion form factor and weakly depends on interaction model of quark in pion.

3. Wigner relativistic SR is responsible for the contribution of the quark magnetic moments to the charge pion form factor.

So, our approach developed in the frame of the instant for RHD, demonstrates of SR not only at $Q^2 \to \infty$ but in the region of low and intermediate Q^2, too.

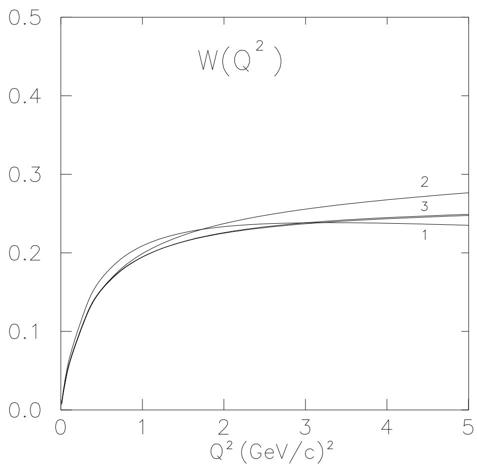

Figure 1: The contribution of the Wigner spin rotation to the electromagnetic form factor of pion at intermediate momentum transfers. The function $W(Q^2)$ is determined by eq. (31). The model wave functions (5)–(7) are used for the calculations. 1. the model (5), 2. the model (6), $n = 2$, 3. the models (6), $n = 3$ and (7).

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